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Interview

Developing and implementing the UN's probabilistic population projections as a milestone for Bayesian demography: An interview with Adrian Raftery

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Abstract

BACKGROUND
Population projections for all countries are published by the United Nations Population Division (UNPD) every two years as part of the World Population Prospects (WPP). Since 2015, probabilistic population projections have been published as part of WPP, produced using Bayesian statistical models. Central to this methodological change was a team of statisticians at the University of Washington, led by Professor Adrian Raftery.

OBJECTIVE
This interview with Adrian Raftery details the history of the UNPD WPP probabilistic population projections, including how the project started, the methodological challenges, main takeaways and lessons, and priorities for future research.

CONTRIBUTION
This interview contributes to the record of scientific thought and the advancement of methodology in demographic research. It demonstrates the evolution of a successful scientific project with large scientific impact and a broader influence on the field of Bayesian demography.

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From the Editor

We inaugurate the 51st volume of *Demographic Research* and mark the 25 years of the journal with an anniversary interview: Professor Adrian E. Raftery from the University of Washington in Seattle discusses the applied side of Bayesian demography in conversation with Monica Alexander. The interview focuses on the United Nations Population Division’s work on global probabilistic population projections, which Professor Raftery has led from the academic side. With this interview, we celebrate one of the finest examples of real-world impact of methodological innovation in population science – some of which was published in *Demographic Research*. Enjoy reading!

Jakub Bijak, Editor of *Demographic Research*, 2018–2024

1. Introduction

Knowing the demographic makeup of a country’s population, and how populations are likely to change in future, is fundamental for government policy and resource planning purposes. Since the 1940s, not long after the time that the United Nations was established, the United Nations Population Division (UNPD) has had a central goal to produce estimates of past populations and projections of future populations for all countries worldwide. These estimates are published as part of the World Population Prospects (WPP), which have been updated approximately biennially since 1950 (Buettner 2020). As even the most vital demographic information is not available in some countries, and the quality of data varies substantially across time and space, reconstruction and projection of populations is not always straightforward, and may require the use of models, demographic or otherwise. The 2015 edition of the WPP represented a substantial shift in the UNPD’s methodology for population projections, which has been carried through to recent WPP editions. Specifically, in WPP 2015, methods were changed to produce probabilistic projections which can be reported with different levels of stochastic uncertainty, moving away from the traditional scenario-based, deterministic populations.

Central to this change was the collaboration between UNPD and a group of statisticians at the University of Washington, led by Professor Adrian E. Raftery, who is the Blumstein–Jordan Professor of Statistics and Sociology, and Adjunct Professor of Atmospheric Sciences at Washington. This interview with Professor Raftery, conducted early in 2024, focused on the history of the UNPD WPP probabilistic population projections, including how the project started, the methodological challenges, main takeaways and lessons, and priorities for future research.

2. Beginnings

MA: ***As a statistician, what was your pathway to getting interested in problems related to demography, and in particular, population projections?***

AR: I had always been interested in population problems, and had some exposure to actuarial science, but had never studied demography in a formal sense. I joined the statistics department at the University of Washington in 1985, a university which was very strong in the environmental sciences. I had the opportunity to be involved in projects on the population dynamics of bowhead whales working with the International Whaling Commission, which led to the development of methods to estimate probabilistic outcomes from deterministic population models (Raftery, Givens, and Zeh 1995[3]). The population dynamics model for bowhead whales is a special case of a Leslie Matrix projection model (Leslie 1945), with expressions for age-specific fertility and mortality rates that are relevant to whale populations. The context and questions being asked naturally led to a Bayesian approach, with the combination of information from the deterministic model and prior distributions combined using a Bayesian melding approach (Poole and Raftery 2000).

While pertaining to whale population dynamics, the 1995 paper received a comment from Shripad Tuljapurkar and Ronald Lee, two prominent researchers in demography, who discussed parallels and applications to human populations (Tuljapurkar and Lee 1995). Lee and Tuljapurkar had been working on probabilistic models for population projection (Lee and Tuljapurkar 1994). I stopped working on whales in 1998, and, while not directly related to human populations, this was the start of working on population dynamics.

3. Model development and collaboration with the United Nations

MA: ***How did the collaboration with the United Nations begin?***

AR: In July 2004, Thomas Buettner, the then Assistant Director of the UNPD, contacted me to discuss improving the methodology around population projections in the WPP. At the time, projections were done using a scenario-based approach, and Buettner was interested in improving the estimation process and incorporation of uncertainty. Buettner had approached various demographers, including Shripad Tuljapurkar, who had suggested my name – in essence, "he's done it for whales, perhaps he could do it for

[3] References and footnotes added by Monica Alexander.

humans". Buettner's first problem that he wanted to solve was population reconstruction, that is, back projection.

From 2004 to 2006, along with Sam Clark (now a Professor in Sociology at Ohio State University), we had intermittent discussions with Buettner while writing a National Institutes of Health (NIH) grant proposal, for which Buettner wrote a letter of support. The proposal was funded in 2006. The proposal centered around probabilistic projections but also included epidemiological and demographic models. The proposal had a lot of ideas, many of which ended up not being pursued, but it did lead to the application of Bayesian melding to estimate uncertainty in national HIV prevalence estimates (Alkema, Raftery, and Clark 2007), which is used by UNAIDS.

MA: ***What did the process of model development look like?***

AR: In terms of working on probabilistic population projections, the period from 2006 to 2009 was really when the methodological development happened. We tried a bunch of different approaches before landing on the final methodology. There was a lot of interaction with UNPD during model development; Patrick Gerland (now Chief of the Population Estimates and Projections Section at UNPD) was the main contact and led the project, with a lot of support from Thomas Buettner and Hania Zlotnik, who was the Chief of UNPD at the time.

The initial uncertainty assessment was focused on fertility, which the UN was prioritizing. Thomas Buettner had introduced the idea of the double logistic model, which, in general, fitted the evolution of the total fertility rate (TFR) very well. In particular, the double logistic model expresses the rate of fertility decline in a population with two conjoined logistic curves, one expressing an increasing rate of decline, and one expressing a decreasing rate of decline. At the time, the UN were using the double logistic model in a scenario-based approach, with five possible scenarios (e.g., fast–fast, fast–slow), and the appropriate scenario for a particular country being chosen based on available data.

With Leontine Alkema (then PhD student, now Professor of Biostatistics at the University of Massachusetts, Amherst) we realized that, rather than taking a deterministic approach, the parameters of the double logistic curve could be estimated in a statistical model (see Alkema et al. [2011] and Alkema et al. [2012] for details). The use of a Bayesian hierarchical model, which was a well-established modeling technique in other fields, was motivated by the data situation, with vastly varying amounts of data being available by country.

MA: ***How were the initial estimates received?***

AR: In 2009 a method was ready for fertility, and the UN organized an expert group meeting in December of that year to discuss the method. There were six leading demographers reviewing the method (including John Bongaarts and Nico Keilman), as well as country-specific experts. There was pretty unanimous support for the method, with a general feeling that it was an improvement on the existing approach taken. For the 2010 revision of the WPP, the UNPD did projections based on the Bayesian method, and used those as the median projection (in the deterministic sense), so they didn't go fully probabilistic, but 'on the side' they released probabilistic fertility projections. This allowed the demographic community to evaluate the new method and respond more broadly.

In the Bayesian hierarchical model, there is a need for a 'global mean' of long-run ultimate fertility rate, around which countries would eventually fluctuate. We set this at 2.1 births per woman, which seemed reasonable given its link to replacement level fertility. But it ended up that this led to an increase in fertility projections in many countries. These increases were somewhat criticized by the demographic community when WPP 2010 was released, and so we had to go back to the drawing board to work out what to do. We ended up extending the model to estimate an eventual fertility level for every country, within the hierarchical framework to deal with data sparsity. Interestingly, before the Bayesian method, the deterministic assumption for fertility was that all countries would converge to the same fertility level, which was 1.85 births per woman. It turned out that, the estimated global mean of eventual fertility in the Bayesian hierarchical model was also 1.85! This really speaks to the strong intuition of demographers. Note that the original *Demography* paper that describes the TFR projection method uses a global mean of 2.1; the updated version of the model, as well as a brief history, is described in Raftery, Alkema, and Gerland (2014).

The new version of the model was the basis of the WPP 2012 release, but projections were still not fully probabilistic. We then worked on a method for producing probabilistic projections of mortality, and these models became the basis of the 2015 edition of WPP. In that year, the scenario-based high–medium–low variants were essentially replaced by the probabilistic intervals. The method for producing probabilistic projections of life expectancy (Raftery et al. 2013; Raftery, Lalic, and Gerland 2014) is similar to the method for the total fertility rate, in the sense that it considers changes in life expectancy over time as modeled with a double logistic curve. The parameters of this model are then estimated within a Bayesian hierarchical framework.

4. Modeling decisions and contributions

MA: ***Aggregate indicators of fertility and mortality are modeled, not age-specific rates. Why?***

AR: The decision to model TFR and life expectancy rather than age-specific rates was partly driven by existing approaches (that is, the use of the double logistic curves to capture fertility and mortality transitions over time), but also driven by the structural patterns in these outcomes: as Lee and Carter (1992) showed us, you can capture the vast majority of variation over time with just a few parameters, and it is much easier to estimate and forecast a smaller number of parameters, in our case to describe the double logistic curves, rather than a full set of separate age-specific rates. The process to obtain age-specific rates from the aggregate indicators is described in Ševčíková et al. (2016). For mortality disaggregation involves fitting the Lee–Carter model and variants (Li and Lee 2005; Lee and Miller 2001). The method for fertility disaggregation calculates proportionate age-specific fertility rates assuming some convergence to a distribution.

MA: ***Why a Bayesian hierarchical model?***

AR: The use of a hierarchical model is really essential in this context, because of the varying amounts of information available. For example, the double logistic curve explains the transition of fertility from relatively high to relatively low levels, and as some countries had only just started the fertility transition, there was not much information available in those populations. Building a model that allowed for some information exchange across countries and shrinkage towards some common mean is extremely valuable. There's also huge variation across countries in the amount of data available over time, and so the hierarchical set up allows for data-sparse countries to be partially informed by patterns in fertility change in other countries.

The Bayesian piece was useful for a number of reasons, including to be able to set demographically-informed priors, which we did in a number of cases, both through being informed by previous literature, and prior elicitation from staff at UNPD. For example, for the mortality model, we needed to set an 'eventual level' of life expectancy. There's an asymptote in the model, which describes the eventual stable increase in life expectancy, and we used research by Nadine Ouelette and others (Ouellette, Barbieri, and Wilmoth 2014) to set an upper bound on how fast that could be. Anther strength of the Bayesian approach was that Markov Chain Monte Carlo algorithms were very useful in fitting nonlinear models in a computationally identifiable way. Additionally, the fact that the model produces a set of trajectories is useful for practitioners to take and propagate uncertainty in their own processes. Finally, using likelihood-based approaches, such as maximum likelihood estimation may have been possible, but usually the assumptions of these methods rest on asymptotics, which may not be appropriate in this context, given we often only have a few data points.

MA: ***How did this work fit into and build upon existing work on population projections?***

AR: One of the questions is, why has the probabilistic approach to demographic projections only emerged so recently? Deterministic projections were well established in demography; the cohort component projection modeling approach dates back to the 1930s (Whelpton 1936), and the US Census Bureau was an early adopter of these methods. Traditionally, any uncertainty in projections is incorporated through subjective scenario-based approaches. At the time, reporting projections with a range of uncertainty was not really expected, but there were increasing calls for this from the 1970s. Nathan Keyfitz called for it in 1972 (Keyfitz 1972), and introduced the ex-post method in 1981 (Keyfitz 1981). Lee and Tuljapurkar presented a time series method (Lee and Tuljapurkar 1994) and then there were expert-based subjective approaches that were prevalent in the 1990s (Lutz, Sanderson, and Scherbov 1998). In the 2000s, there was a project in Europe headed up by Nico Keilman and Juha Alho, which used the ex-post method to produce probabilistic projections for all European countries (Alho et al. 2006). However, it wasn't really adopted by European statistics agencies.[4]

So there were definitely calls to go probabilistic, and some methods around, but no one really implemented them for practical forecasts. The existing methods had not really been tested to produce probabilistic forecasts across a range of contexts. That was the big difference for our project – we were directly working with the UN, and we had to deal with various types of data, and we had to get a method working across the board.

[4] For a nice overview of the literature, see Raftery and Ševčíková (2023).

5. Keys to success

MA: ***Why did this project work?***

AR: In an agency, innovation is often hard. It takes a lot of time, and a lot of work, and incentives are often aligned with promotion, not innovation. However, the UNPD had a number of characteristics that helped to promote innovations. Right from the beginning, there has been a lot of collaboration and interconnectedness with academia: Frank Notestein was the first director, Pascal Whelpton was the second, people like Sam Preston and Patrick Heuveline have worked there, and then of course most recently John Wilmoth is the director. This means that there's a culture of intellectual curiosity and willingness to understand and adopt new methods. Secondly, the people who work there are highly selected: Many are motivated by wanting to improve society and the state of the world, and as such the staff were really committed to the overall goal, in spite of relatively limited resources.

A large part of the success of this project, was also due to the availability of accessible, easy to run software. Initial software development was student-led, until Hana Ševčíková led development in 2009. The methodology for population projections can be run through R using the `bayesPop` package, which uses outputs of fertility and mortality projections from the `bayesTFR` and `bayesLife` packages, respectively.

Software was really important in terms of being able to communicate with the UN team, and to investigate questions and issues. It enabled the UN group to play around with results and explore sensitivities to model choices. This, for example, led to a new set of priors being placed on the double logistic curve for life expectancy. The double logistic curve is described by six parameters, which can be manually changed and updated through the `bayesLife` package. Staff from the UN side were doing this to explore changes in life expectancy projections, and we were able to see what range of manual values they had edited for particular countries. We subsequently used this information to update the priors of the six parameters for that subset of countries. This process is described in a UN technical report, led by Helena Cruz Castanheira (Castanheira, Pelletier, and Ribeiro 2017), and was used to improve estimates for the 2017 revision of WPP. It's a great example of prior elicitation working well, as practitioners have good intuition about outputs, but not statistical parameters, but in this case, we could convert the information to be translated to the parameter space.

The software piece of this project, and the emphasis on producing a tool that is usable and understandable for practitioners is a lesson in the importance of open science and reproducibility. Although the topic of open science is much more prevalent now, this project was ahead of its time, having devoted resources to software development from its inception in 2006.

6. Practical lessons

MA: ***This is a project where academics and practitioners were explicitly working together. How did the practical nature of the objective shape how you approached the problem?***

AR: The goal for me is to always do good science. And you do better science when you have clear goals. For one thing, you've got an existence proof of usefulness. Often problems that occur in some contexts come up elsewhere. And often interesting mathematical problems come out of practical problems. In the demography case, there's a question of how to deal with the 'law of conservation' of demography (i.e., the demographic accounting identity). This is particularly important in demographic reconstruction, and has implications for the uncertainty of estimates.[5]

I've never thought of it as being 'under time pressure'; I still always have a goal of producing good work. But certainly the impact of this work would have been far less if it did not go hand in hand with the practical goal. And this practical goal also improved the quality of the work: Instead of incrementally improving on a model in the academic literature, we are working on a problem and constantly getting feedback from practitioners who are always questioning and bringing up issues. So you have to think out of the box to solve these real problems.

A large part of this project was also making this complex methodology more accessible and usable to a broader audience. As well as the software mentioned previously, we have run a lot of short courses on how to use the software to obtain projections. The first one was in Rostock in 2014, and we've done a number of short courses at population conferences, for example, at the Population Association of America (PAA) annual meeting, the International Union for the Scientific Study of Population (IUSSP) conference, at the Latin American Population Association (ALAP) congress (which included 60 participants from 33 countries!), and at other institutions all over the world.

MA: ***How do we think about and communicate uncertainty in projections?***

AR: One of the questions you have to ask is, why are we even producing these projections in the first place? The answer is really for policy making and decision making. For example, when one of the earlier probabilistic WPP set of estimates came out, the projected population increase in Nigeria was influential in resource allocation in building more elementary schools. Of course, the uncertainty aspect is also important here, because you want to make sure that there are enough schools for the future population of

[5] See for example, Wheldon et al. (2013).

children. But this is a different degree of understanding uncertainty than the quantification of uncertainty in the scientific sense. The use and communication of uncertainty really depends on the context.[6]

MA: ***Student trainees were a large part of this project. How has this project shaped how you approach student mentoring in future?***

AR: Students were central to this project; indeed the majority of the NIH grant funding was allocated to PhD students. I count myself very lucky to have had very good students. Six PhD students have graduated on the project, and every student has played a very important role, with most papers having students as first authors. In terms of the structure of the workflow, I met with students individually once a week, but then once a month we would have group meetings to give short updates and discuss ideas. We also have a Friday morning working group (on Applied, Bayesian and Computational Statistics [ABC], which started in 1994), which was another great way of getting together and sharing ideas. I think having multiple PhD students at one time creates an environment of mutual learning and rapid exchange of ideas.

7. The future of Bayesian demography

MA: ***What are the future priorities for demographic estimation and forecasting in general?***

AR: Better treatment of migration is a big priority. Demographic modeling research has traditionally focused on mortality, then fertility, then migration, but really the priorities in modeling should now be the reverse. There is room for improvement in both modeling (in particular, in- and out-flows by age), and also in data collection. Traditionally data on migration has been pretty unreliable, but more recently there have been other potential data sources (such as social media and administrative data) that show promise.

In terms of projections, migration is important because for the majority of countries, it is the source of the most uncertainty about future populations. For example, in Germany, about 74% of uncertainty is due to migration. We have developed methods for probabilistic projections for net migration (Azose and Raftery 2015), but they haven't been incorporated into WPP yet (although there are plans).

Another priority includes the better treatment of the errors and uncertainty around data sources that are inputs to projection models. We've done some work on this with the

[6] For more discussion of these ideas see Raftery (2016).

fertility model (Liu and Raftery 2020), which the UN is now using,[7] but there are other sources of uncertainty in population counts from censuses, for example, that are not taken into account.

Improving subnational population estimation and projection is another priority for the future. This is both in terms of geographic area but also by subgroups of the populations. There's interest in not only geographic administrative areas, but also at a more fine-grained scale, for example the WorldPop group at the University of Southampton (Tatem 2017). There's a lot of scope to investigate ways of incorporating the WorldPop estimates into population projections.

A final question to be thinking about is how can new methods from statistics and machine learning potentially contribute to problems in demography? There's some initial work using machine learning methods in population projections, but this area is still in its infancy.

MA: ***How do you see the different groups who do population projections fitting together?***

AR: The thing that sets the UNPD apart is that it's a sustained effort to produce population projections. Other groups have produced sets of estimates at various points in time (for example, the International Institute for Applied Systems Analysis [IIASA], the Institute for Health Metrics and Evaluation [IHME], the US Census Bureau), but the UN has a mandate to produce regular sets of estimates, has unprecedented access to data, and has been driving to improve methodologies and access to inputs, modeling code, and outputs. It's healthy to have alternative methodologies, constructive discussions, and push back, but it seems like the UN will continue to be a major player in the space going forward.

MA: ***Bayesian demography, even with its long history, has been gaining rapid momentum especially since WPP 2015. Where do you see these methods developing in the future?***

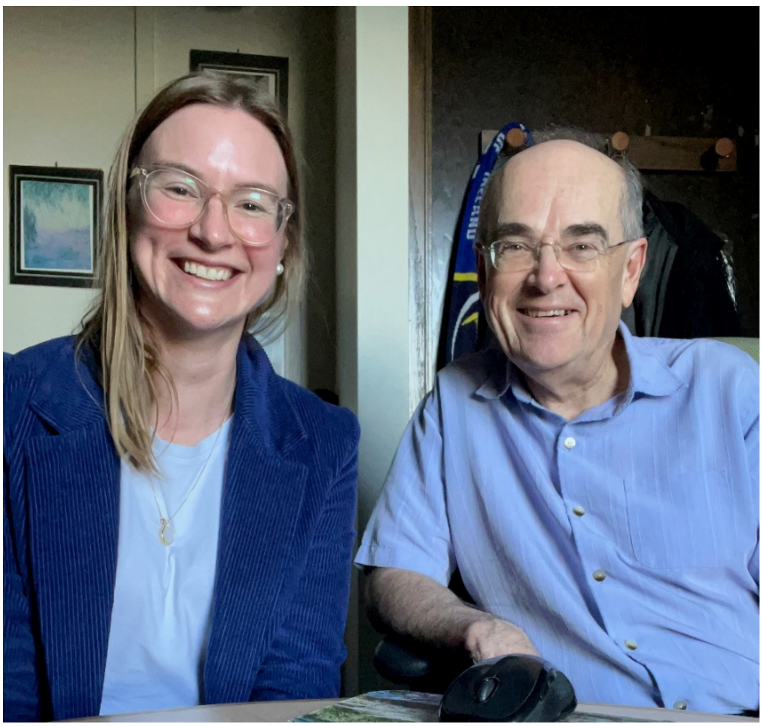
AR: The use of Bayesian methods helped solve many issues that were present in traditional deterministic models. Part of this was being 'Bayesian', but part of this was just being statistical. But there are reasons to believe that the Bayesian paradigm will continue to be important. For instance, population forecasting will always be a central part of demography, because of its importance in policy making, and the Bayesian

[7] There is also other work along these lines in the estimation of child mortality, e.g., Alkema and New (2014).

approaches generate probabilistic outputs which are important and can be used to answer all types of questions.[8]

But it's important to keep an eye out for new things: the evolution of statistics is driven by the appearance of new kinds of data that generate questions that require new forms of analysis. There are new forms of data in demography; for example, in the migration case, we have various new, imperfect sources of information, and this is important to try and use, because of how little we know about migration. Bayesian methods seem suited to this task – the need to combine information and borrow strength – but there are many more questions to be answered.

MA: ***On behalf of*** **Demographic Research*****, thank you so much for this insightful interview, and congratulations and thank you for all your contributions to the WPP project and to the broader field of Bayesian and statistical demography.***



(Monica and Adrian at the University of Washington, 22 February 2024)

[8] See also Bijak and Bryant (2016).

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