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Research Article

Modeling fertility curves in Africa

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Modeling fertility curves in Africa

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Abstract

The modeling of fertility patterns is an essential method researchers use to understand world-wide population patterns. Various types of fertility models have been reported in the literature to capture the patterns specific to developed countries. While much effort has been put into reducing fertility rates in Africa, models which describe the fertility patterns have not been adequately described. This article presents a flexible parametric model that can adequately capture the varying patterns of the age-specific fertility curves of African countries. The model has parameters that are interpretable in terms of demographic indices. The performance of this model was compared with other commonly used models and Akaike's Information Criterion was used for selecting the model with best fit. The presented model was able to reproduce the empirical fertility data of 11 out of 15 countries better than the other models considered.

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1. Introduction

Parametric and non parametric models have been reported to have useful applications in demographic research. Apart from being useful when creating hypothetical rate schedules in forecasting and projection, they also serve to condense complex data into smaller indices (Schmertmann 2003; Peristera and Kostaki 2007). Several models have; therefore, been proposed to model fertility as the major determinant (of the three demographic variables namely, fertility, mortality, and migration,); of the size and structure of any population. These models have been commonly created for the developed countries of the world and usually fit excellently the population they are intended to model (Hoem et al. 1981).

It is pertinent, however, to mention that though there are many fertility models in the literature, few have been specifically generated to describe age-specific fertility patterns in Africa; despite the fact that most governments of the sub Saharan African countries are targeting lower total fertility rates to meet the Millennium Development Goals (MDGs) (United Nations 2000). To make reaching these targets possible, a better understanding of the current pattern of age specific fertility rate (ASFR) of African countries is required. Mathematical models, when well constructed, can aid in this understanding as they provide better insight into some characteristics of the distributional pattern of fertility in Africa. The goal of any modeling exercise is to extract as much information as possible from available data and to provide an accurate representation of both the known and unknown aspects of the phenomenon being studied (Salomon and Murray 2001). Modeling fertility in Africa has also become necessary to enable a meaningful comparison of fertility across the countries in the region in the face of the current fertility transition. Already, fertility can be compared using a wide variety of existing conventional measures, summary indices, or averages that are commonly reported for fertility data. These include total fertility rate (TFR), general fertility rate, and the crude birth rate. Few comparisons, however, are made based upon the detailed distribution of the age-specific fertility curve. Not all information in the curve can be conveyed by these summary indices. There is still much to be described in terms of the variance, skew, kurtosis, and symmetry of fertility distributions for individual countries on the continent.

In this article, we propose a mathematical model for ASFR using the complementary error function (defined below). The proposed model is a flexible one that can capture various shapes of ASFR. It also provides a mathematical description of some fertility indices through its interpretable parameters. The efficacy of the model was determined by comparing its performance with other fertility models.

The age pattern of fertility in Africa is described in the next section. In Section 3, we provide a brief review of some existing models for fertility patterns and then propose our model in Section 4. The results of fitting our model as well as other models to the fertility data are presented in Section 5. This paper concludes with a discussion of the implications of our findings.

2. The pattern of age-specific fertility rate for African countries

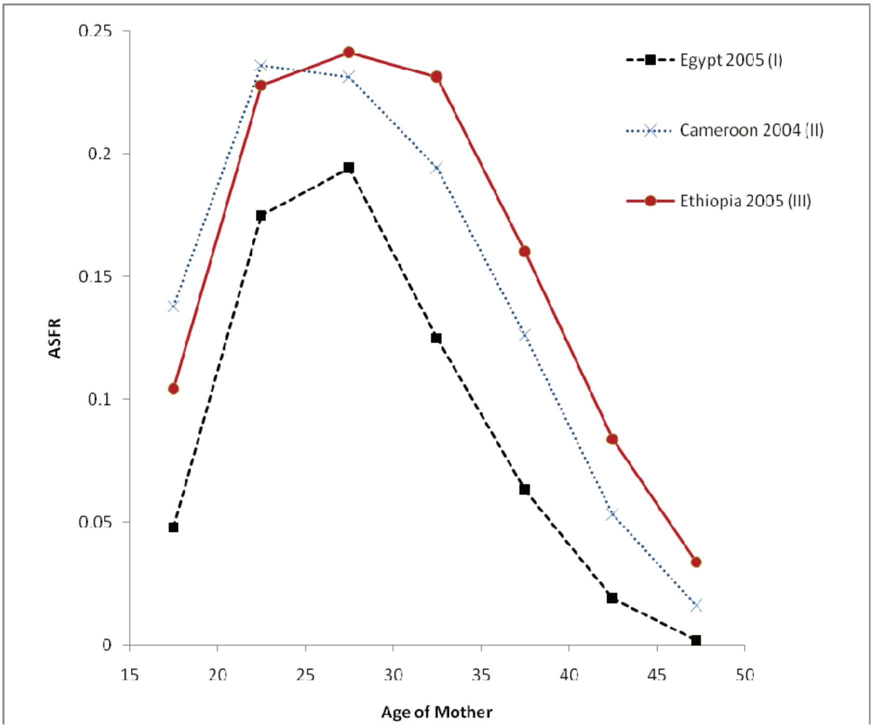
African countries are numbered among those in the world with the highest fertility rates (Norville, Gomez, and Brown 2003). Several factors have contributed to these high levels of fertility. Some of these factors include high level of infant and child mortality, early and universal marriage; resulting in child bearing which begins early and continues for much of the reproductive life span. Additionally, the low use of contraceptives and the high value societies place on childbearing contributes to high fertility among the populations of African countries. As a result of the perceived high infant and child mortality, people have many children in the hope that some children will survive to carry on the family line. The institution of polygamy promotes competition among co-wives to have children; which also contributes to high fertility in many African countries. Furthermore, the traditionally high value placed on marriage ensures not only its universality, but also its occurrence early in life. The consequence is that child-bearing will start early and in most cases, continue until late in the reproductive lifespan. Caldwell (1980) asserted that one explanation for high fertility rates is that the lifestyle of most of the population in African countries is commonly rural and less industrialized than that of the rest of the world. This rural lifestyle encourages the population to engage predominately in subsistence farming, and as a result, to consider large family size as an asset.

Makinde-Adebusoye (2001) classified African countries into three categories based on their positions on a demographic transition continuum. Those countries classed as Category I are those that have fertility data supporting a demographic transition from high to much lower fertility levels. Countries that fall into this category include Cote D'Ivoire, Ghana, Nigeria, Kenya, Rwanda, Botswana, Zambia, Zimbabwe, Egypt, Morocco, and Tunisia. Countries in category II are those that have recorded a small decline in TFR. These countries are Benin, Mauritania, Senegal, Cameroon, Central Africa Republic, Malawi, Tanzania, and Swaziland.

Category III countries are those in which the fertility levels appear to have stabilized around a peak with TFR of approximately 6 or more. Countries in this category are Burkina Faso, Liberia, Mali, Togo, Burundi, Ethiopia, Madagascar, Mozambique, and Uganda. Others included in this category are Niger, Angola, and the Democratic Republic of Congo. It is worth noting that the above categorization is not exhaustive for all African countries.

A plot of the ASFR for countries from each of the above categories reveals some differences in the age pattern of fertility among African countries. Figure 1 shows the five-year age pattern of fertility for Egypt, Cameroon, and Ethiopia (chosen from categories I, II, and III respectively). Some characteristics of African fertility pattern can be observed from Figure 1. The pattern for Egypt starts with a lower rate than those of the other two countries. It rises to reach the peak in the late twenties (age group 25-29) before it starts dropping rapidly. Unlike Egypt, Cameroon's pattern starts with a relatively higher rate and peaks in the early twenties (age group 20-24). It does not demonstrate a rapid drop after the peak. In contrast to the other two countries, Ethiopia depicts a broad peak shape that extends from the early twenties to the early thirties before a rapid drop. Additionally, whereas the tail of the curve almost approximates zero in the early forties for Egypt, that of Ethiopia is still much higher at the same point.

Figure 1: Age specific fertility curves for some selected African countries



These features are summarized for each category below:

Category I (example: Egypt)
Low f15
Peak in late 20s
Rapid drop after late 20s
Very low rates in 40s

Category II (example: Cameroon)
High f15
Peak in early 20s
Rate of dropping after the peak not rapid

Category III (example: Ethiopia)
High f15
Broad peak that extends from early 20s to early 30s
Rapid drop after the early 30s
Very high rates in 40s

These features are commonly exhibited in the fertility pattern of African countries, particularly those of categories II and III, though they may not be exhibited in the fertility pattern of the developed countries. Since many developed countries have TFR below the replacement level, it is necessary to have new, more suitable parametric model(s) to study the differing fertility patterns of African countries.

Notwithstanding the various patterns noticed in the age-specific fertility curves of African countries, the continent, like other parts of the world is experiencing a fertility transition. This transition, however, is very different from those experienced in other places with similar levels of fertility (e.g., Asia and the Middle East). Unlike Africa, in some Asian countries, there are strong proscriptions against female premarital sexual activity and the great majority of brides are probably virgins (Caldwell, Orubuloye, and Caldwell 1992). The African sexual pattern is different. It is characterized by widespread premarital sexual relationship for both females and males. In fact, about four-fifths of urban women and two-thirds of rural women that were studied claimed to not have been virgins at the time of marriage (Orubuloye, Caldwell, and Caldwell 1991). Studies by Caldwell, Orubuloye, and Caldwell (1992) have also pointed out that there is similarity in contraceptive usage and fertility decline at all ages, both inside and outside marriage in Africa and that this pattern presents a new type of demographic transition to the world. In Asia and Europe, on the contrary, fertility declines are usually nonexistent or very moderate, below age 25. The decline starts gradually and increases with age reaching a significant 30% after the age of 40. Therefore, age-specific fertility curves in Africa, are more likely to be different from those found outside Africa and hence, fertility models developed and adequately used for the latter are likely to be insufficient for the former.

3. Models for fertility schedules

Various types of models have been proposed for modeling the one-year age specific fertility curves of many populations, but fitting these models to curves of African populations, with the attendant high-fertility levels, has not been sufficiently undertaken. Among them are the Coale-Trussell function (Coale and Trussell 1974, 1978); the Pearson Type 1 (Mitra 1967; Romaniuk 1973); Type III curves (Nurul Islam and Mallick 1987); the Beta and Gamma functions (Hoem et al. 1981); the Hadwiger function (Hadwiger 1940; Gilje 1969); and the cubic splines model (Hoem and Rennermalm 1978; Gilks, 1986).

Others are the Brass procedures (Brass 1974, 1979); the Gompertz curve (Wunsch 1966; Murphy and Nagnur 1972; Fraid 1973); the polynomial models (Brass 1960); and the quadratic spline function by Schmertmann (2003). Islam and Ali (2004) proposed a bi-quadratic polynomial model for the ASFR of a rural area of Bangladesh. Considering the recent increase noticed in the fertility rate of young women in developed countries such as the UK, Ireland and the US, Peristera and Kostaki (2007) proposed two models (which they refer to as Model 1 and Model 2, respectively) to model the pattern. All these models have been found to provide excellent fit for the one-year age-specific fertility distributions of populations with natural fertility, transitional fertility, and controlled fertility. A detailed descriptions of the mathematical formulae of most of the models can be found in Hoem et al. (1981) and Peristera and Kostaki (2007).

It might be possible to fit these models to African fertility data but the results obtained may not be very accurate. A quick look at the individual models reveals the reason for this lack of accuracy. The Coale-Trussell function can be considered unsuitable because of its orientation to fertility control. It focuses on marital fertility; the parameters determining the shape of the curve are age of marriage, proportion marrying, and the degree of fertility control (Chandola, Coleman, and Hiorns 1999). The contribution of non-marital fertility to the current fertility patterns in Africa cannot be overlooked in any fertility study in the region. The computation required by the function can also be daunting. Nonetheless, the Coale-Trussell function yielded good results when applied to a fertility schedule by Hoem et al. (1981), and Schmertmann (2003). To determine the age distribution of fertility using the Pearson Type 1 curve, values for the average age, variance, asymmetry, and kurtosis are needed which are not often provided with the other fertility data of African countries. Some simplifications of the curve, however, enable estimates to be made from the mean and modal ages of fertility. Most work on this distribution has been devoted to the projection of fertility and future birth, as is the case with Gompertz models, which also focuses on marital fertility through its parameters (Romaniuk 1973).

Polynomial and spline models when elevated to a suitable degree can be made to fit fertility curves well, but the lack of meaningful parameters makes them ill-suited for comparative purposes, and as a result, their use has been limited. Unmodified gamma and beta distributions may not generate curves of appropriate shape for African fertility. However, the modified versions of these distributions by Hoem et al. (1981) were found to be useful for fitting Danish fertility curves. Also, studies have revealed that the Hadwiger function is more suited for smooth fertility curves (see Chandola, Coleman, and Hiorns 1999), and that it has a common problem of overestimating fertility data towards the end of the reproductive age (Hoem et al. 1981). Hence, it may not be appropriate for fitting African fertility curves. The Quadratic spline model by Schmertmann (2003) might be more appropriate, but using the model requires the use of an author provided program (Schmertmann 2005); which is accessible via the internet. Without this program estimating the parameters can be awkward, unfortunately, using the internet for this purpose can be challenging for some researchers in the African region who may lack sufficient internet access. These researchers would prefer using more accessible, alternative models when available.

To accurately model fertility patterns in Africa, a new mathematical model that is both easily used, and provides good fit for the data is required. Such a model could reveal important parameters which need to be taken into account when comparing fertility between countries and across time. This undoubtedly would increase our understanding of fertility pattern in the region.

4. The model

The model we present in this work is a two-component model with the second component being a constant term. Chandola, Coleman, and Hiorns (1999) first considered the possibility of incorporating a separate term into the Hadwiger functions to handle excess early-age fertility. Similar work was done by Peristera and Kostaki (2007) in Model 2 when modeling the excess early age fertility of the UK, Ireland and the US.

We present the following model for the ASFR pattern. The model uses the error function and will therefore be referred to as the *Adjusted Error Model* (AEM). It is given by

$$f\left(x\middle|a,b,\mu,s_1,s_2\right)=a+b\cdot\begin{cases} Erfc\left[\left(\dfrac{x-\mu}{s_1}\right)^2\right] & \text{if } x\le\mu \\ Erfc\left[\left(\dfrac{x-\mu}{s_2}\right)^2\right] & \text{if } x>\mu \end{cases} \tag{4.1}$$

where *Erfc()* is the complementary error function and is defined as

$$Erfc\ (z)=\frac{2}{\sqrt{\pi}}\int_{z}^{\infty}e^{-t^2}\,dt \tag{4.2}$$

f(x) is the ASFR at age *x* of the mother, *b* is the modal fertility rate μ is the mean age of motherhood while s_1 and s_2 are the spread of the distribution of the age at their respective points of reference. The constant *a* approximates the fertility rate in the age group 45-49.

The parameters of the proposed model were estimated by minimizing the following sum of square residual (SSQ):

$$\sum_x\left(f_x-\hat{f}_x\right)^2$$

where f_x is the empirical fertility rate at age *x* and $\hat{f}_x$ is the estimate of f_x.

It is common when modeling fertility, and it is required to determine the performances of a number of models, to plot the predicted values of the models alongside the observed curve and then to pick the model that has the best fit; if any model stands out as obviously better than the others. Alternatively, SSQ for all the models will be computed and the model with least SSQ will be preferred. These methods cannot be said to have balanced both model fit and model complexity caused by the addition of more parameters. In this paper, we have chosen to use the Akaike's Information Criterion (AIC) (Akaike 1974) displayed in (4.3) as a measure of selecting model with best fit among the models that have been fitted to the fertility data of African countries alongside the proposed AEM. The AIC is a criterion for comparing alternative functions by adjusting the SSQ for the number of observations and the number of parameters in the model. The criterion can be used to decide if the improved fit is worth the decreased degrees of freedom and the increased complexity of the function caused by the addition of another parameter to a model.

The variant of the AIC used in this paper is given by

$$AIC=2k+n\ln\left[\frac{SSQ}{n-k}\right] \tag{4.3}$$

where *k* is the number of parameters in the model and *n* is the number of observations. When comparing the performances of a number of models, the model with least AIC is usually preferred as the best. The AIC procedure has been discussed and successfully used to identify model with best fit by Akaike (1977), Tong (1977), Ozaki (1977), Larimore and Mehra (1985), and Koehler and Murphree (1988), among others.

5. Evaluation of the model

In order to determine the performance of the proposed model, we fit it alongside five other commonly used models namely; the Quadratic spline (QS), Beta function, Gamma function, Hadwiger function, Model 2, and to the empirical five-year fertility data of 15 African countries. Age-specific fertility datasets for these countries are based on a five-year fertility instead of single-year age. Hence, all analyses are based on a five-year age. The data for all countries were obtained directly from published reports of Demographic and Health Survey (DHS) (available online from ORC Macro's website http://www.measuredhs.com). For the purposes of this study, the following countries were included in the analyses: Burkina Faso (2003), Cameroon (2004), Chad (2004), Ethiopia (2005), Egypt (2005), Ghana (2003), Kenya (2003), Morocco (2003-04), Namibia (2000), and Niger (2006). Additionally, Nigeria (2003), Rwanda (2005), Senegal (2005), South Africa (2003), and Zambia (2000-01) were included. The latest available reports for these countries at the time of this research were used. The parameter estimates for the models were obtained by minimizing the SSQ. This was accomplished with the aid of the inbuilt solver in MS Excel. Luckily, the complementary error function used in proposing the AEM is one of the inbuilt functions in MS Excel. The Quadratic spline estimates were obtained using the program provided by Schmertmann (2005) (available at the web page http://www.demographic-research.org/Volumes/Vol12/5/qsfit/qsfit.html).

Tables 1 to 15 in Appendix A present the results of the estimates. The Tables provide the empirical fertility data for each of the countries, the fitted values from each of the six models, the values of the SSQ, the number of parameters, and the values of the AIC. The least AIC value for each case is written in bold case. We also present the parameter estimates of the AEM for selected countries in Table 16. These show the nature of the estimates for each of the model's parameters. Moreover, in Appendix B, the plots of the empirical and fitted values of the models for three countries are presented in Figures 2 to 4. Figure 5 presents the relationship between parameter b of the AEM and the modal fertility rate while the relationship between parameter a and the fertility rate at the age group 45-49 is presented in Figures 6.

Results from Tables 1 to 15 reveal that the values of the AIC for the AEM is lowest for 11 of the 15 countries considered. This shows that this model is better able to reproduce the empirical fertility data of African countries than the other existing models considered. The QS has the least AIC values for 3 of the countries, namely, Egypt, South Africa and Zambia while the Hadwiger model has the lowest for Namibia. The SSQ value of the AEM is lower than that of the QS for Zambia but QS is preferred in reproducing the empirical data by the selection criterion because it has fewer parameters. The same is the case for Namibia where the SSQ of AEM, Beta, Gamma and Model 2 are lower than that of the Hadwiger but the Hadwiger model has the lower AIC because it has fewer parameters.

The strong association between the estimates of the AEM parameters and some features of African fertility allows them to be given demographic interpretations. Some possible interpretations can be explored in Figures 5 and 6. The correlation between the constant parameter a and the fertility rate in the age group 45-49 is found to be significant. Parameter b is strongly associated with the modal fertility rate as evident from Figure 5. The μ parameter is found to relate well with the mean age of motherhood (estimates for selected countries are provided in Table 16) while estimates of s_1 *and* s_2 can best describe the spread of the fertility distribution.

It is worth noting that the AEM is a flexible model that captures the fertility pattern of countries from different parts of Africa well. The model can reproduce the patterns of fertility for those countries whose fertility data demonstrate apparent support for a demographic transition from high to much lower fertility levels; those that have recorded a small decline in TFR, as well as those whose fertility level seems to have stabilized as described by Makinde-Adebusoye (2001). Egypt and Zambia were categorized as countries experiencing a decline from high to much lower fertility. The fertility patterns of these countries are similar to that of South Africa as they were altogether better captured by the QS; one may therefore infer that South Africa also belongs to this category. Another remarkable observation from the results obtained indicates that the Beta model does better than the Gamma for most of the data considered in this study using the AIC as a tool for model choice. This supports the results of Peristera and Kostaki (2007) when both models, among others, were applied to the fertility pattern of “modern population.” Moreover, plots of the empirical and fitted values of the ASFR as displayed in Figures 2, 3, and 4 all reveal that the Hadwiger function overestimated the curves at the peak and towards the end; a problem which has long been noticed with the function by Hoem et al. (1981).

6. Conclusions

Despite the wide range of models proposed for demographic variables in the literature, not much has been specifically put forward to model the ASFR of African countries. In this work, an Adjusted Error model, which is flexible enough to capture the varying fertility patterns of African countries and that has parameters that can be interpreted in terms of demographic indices is presented. The performance of this model alongside some other commonly used models was evaluated using the fertility data of 15 African countries. Using AIC as a model selection criterion, it was determined that the model we propose has the best fit for 11 out of 15 countries. The Quadratic spline has the best fit for 3, while the Hadwiger model captures that of only 1 country better.

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Appendix A

Table 1: Empirical and fitted values for the ASFR for Burkina Faso with values of minimization and model selection criterion

Burkina Faso 2003							
Data		**Model Fit**					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**138**	131	131	137	152	136	135
20-25	**275**	274	275	263	243	261	261
25-30	**271**	274	269	281	284	289	285
30-35	**241**	238	240	242	252	239	243
35-40	**181**	180	187	176	175	166	174
40-45	**106**	108	107	105	97	103	103
45-49	**42**	41	19	45	44	59	46
10^{6}. SSQ		21.8	549	320	1897	1124	474
K		5	4	5	4	3	6
AIC		**-70.0**	-52.2	-51.2	-43.6	-51.2	-41.6

Data source: http://www.measuredhs.com/pubs/pdf/FR154/04Chapitre4.pdf , Table 4.1 p. 48.

Table 2: Empirical and fitted values for the ASFR for Cameroon with values of minimization and model selection criterion

Cameroon 2004							
Data		**Model Fit**					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**138**	138	138	140	144	133	139
20-25	**236**	235	235	229	223	238	233
25-30	**231**	234	236	241	243	242	238
30-35	**194**	192	188	191	194	181	190
35-40	**126**	124	126	119	118	114	123
40-45	**53**	56	60	58	56	64	60
45-49	**16**	15	9	20	21	33	13
10^{6}. SSQ		24.0	158	234	451	856	137
K		5	4	5	4	3	6
AIC		**-69.3**	-61.0	-53.4	-53.6	-53.1	-50.3

Data source: http://www.measuredhs.com/pubs/pdf/FR163/04chapitre04.pdf , Table 4.1 p. 62.

Table 3: Empirical and fitted values for the ASFR for Chad with values of minimization and model selection criterion

Chad 2004							
Data		**Model Fit**					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**187**	188	180	189	189	177	189
20-25	**295**	289	310	285	280	299	286
25-30	**275**	292	291	295	302	301	295
30-35	**263**	246	255	246	245	229	245
35-40	**163**	166	194	165	155	148	165
40-45	**73**	76	107	78	78	86	81
45-49	**13**	12	18	15	32	47	9
10^6. SSQ		636	2700	814	1713	3498	897
K		5	4	5	4	3	6
AIC		**-46.4**	-41.1	-44.6	-44.3	-43.3	-37.1

Data source: http://www.measuredhs.com/pubs/pdf/FR170/04Chapitre04.pdf , Table 4.1 p. 57.

Table 4: Empirical and fitted values for the ASFR for Ethiopia with values of minimization and model selection criterion

Ethiopia 2005							
Data		**Model Fit**					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**104**	105	104	105	117	102	105
20-25	**228**	224	226	223	207	222	224
25-30	**241**	251	248	251	255	260	251
30-35	**231**	221	219	222	230	217	220
35-40	**160**	160	164	160	157	149	160
40-45	**84**	87	91	89	83	90	92
45-49	**34**	33	16	31	35	50	31
10^6. SSQ		226	595	256	847	991	315
K		5	4	5	4	3	6
AIC		**-53.6**	-51.7	-52.7	-49.2	-52.1	-44.4

Data source: http://www.measuredhs.com/pubs/pdf/FR179/FR179.pdf , Table 4.1 p. 47.

Table 5: Empirical and fitted values for the ASFR for Egypt with values of minimization and model selection criterion

Egypt 2005							
Data		Model Fit					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**48**	47	48	50	49	53	67
20-25	**175**	177	175	175	174	170	157
25-30	**194**	187	194	193	195	197	199
30-35	**125**	134	125	128	125	127	135
35-40	**63**	59	62	59	58	57	49
40-45	**19**	15	22	19	22	20	10
45-49	**2**	6	3	4	7	6	1
10^6. SSQ		178	12.7	29.6	62.1	118	1070
K		5	4	5	4	3	6
AIC		-55.3	**-78.6**	-67.8	-67.5	-67.0	-35.9

Data source: http://www.measuredhs.com/pubs/pdf/FR176/04Chapter04.pdf , Table 4.1 p. 44.

Table 6: Empirical and fitted values for the ASFR for Ghana with values of minimization and model selection criterion

Ghana 2003							
Data		Model Fit					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**74**	74	74	74	76	73	73
20-25	**176**	176	176	175	172	174	178
25-30	**210**	209	207	210	213	215	208
30-35	**182**	186	185	186	187	184	185
35-40	**141**	135	139	132	129	127	134
40-45	**70**	74	77	77	76	77	78
45-49	**36**	35	14	35	39	42	32
10^6. SSQ		74.3	568	136	244	302	148
K		5	4	5	4	3	6
AIC		**-61.4**	-52.0	-57.2	-57.9	-60.5	-49.7

Data source: http://www.measuredhs.com/pubs/pdf/FR152/04Chapter4.pdf , Table 4.1 p. 54.

Table 7: Empirical and fitted values for the ASFR for Kenya with values of minimization and model selection criterion

Kenya 2003							
Data		**Model Fit**					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**114**	114	114	118	118	117	116
20-25	**243**	242	242	235	232	234	234
25-30	**231**	235	237	242	248	248	244
30-35	**196**	191	187	191	187	186	191
35-40	**123**	124	126	121	114	113	119
40-45	**55**	56	60	57	59	60	57
45-49	**15**	14	9	16	28	29	16
10^6. SSQ		46.9	179	226	757	829	283
K		5	4	5	4	3	6
AIC		**-64.6**	-60.1	-53.6	-50.0	-53.4	-45.1

Data source: http://www.measuredhs.com/pubs/pdf/FR151/04Chapter04.pdf , Table 4.1 p. 52.

Table 8: Empirical and fitted values for the ASFR for Morocco with values of minimization and model selection criterion

Morocco 2003-2004							
Data		**Model Fit**					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**32**	35	34	33	37	30	31
20-25	**104**	99	99	100	95	99	105
25-30	**124**	132	134	132	135	137	129
30-35	**125**	120	115	119	120	115	115
35-40	**78**	79	76	78	73	71	79
40-45	**28**	29	36	31	33	36	36
45-49	**5**	4	6	4	11	16	0
10^6. SSQ		123	293	123	340	540	223
K		5	4	5	4	3	6
AIC		**-57.9**	-56.6	**-57.9**	-55.6	-56.4	-46.8

Data source: http://www.measuredhs.com/pubs/pdf/FR155/04Chapitre04.pdf , Table 4.1 p. 44.

Table 9: Empirical and fitted values for the ASFR for Namibia with values of minimization and model selection criterion

Namibia 2000							
Data		Model Fit					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**88**	89	78	89	94	86	90
20-25	**166**	164	165	163	154	163	159
25-30	**176**	180	162	179	183	186	181
30-35	**160**	162	155	163	167	160	165
35-40	**137**	126	134	126	123	118	126
40-45	**71**	79	93	80	76	78	79
45-49	**38**	36	34	35	40	49	36
10^6. SSQ		206	830	236	493	637	292
K		5	4	5	4	3	6
AIC		-54.3	-49.3	-53.3	-53.0	**-55.2**	-45.0

Data source: http://www.measuredhs.com/pubs/pdf/FR141/03chapter03-a.pdf , Table 4.1 p. 47.

Table 10: Empirical and fitted values for the ASFR for Niger with values of minimization and model selection criterion

Niger 2006							
Data		Model Fit					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**199**	199	199	200	200	190	201
20-25	**302**	298	301	297	291	306	292
25-30	**296**	305	300	307	314	315	309
30-35	**271**	267	265	266	267	254	268
35-40	**203**	197	203	195	186	179	194
40-45	**105**	112	115	114	110	115	112
45-49	**44**	42	20	41	57	70	42
10^6. SSQ		208	698	325	935	2099	419
K		5	4	5	4	3	6
AIC		**-54.2**	-50.6	-51.1	-48.5	-46.9	-45.0

Data source: http://www.measuredhs.com/pubs/pdf/FR193/04Chapitre04.pdf , Table 4.1 p. 56.

Table 11: Empirical and fitted values for the ASFR for Nigeria with values of minimization and model selection criterion

Nigeria 2003							
Data		**Model Fit**					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**126**	126	127	126	124	108	128
20-25	**229**	230	226	230	227	242	226
25-30	**274**	272	277	273	280	283	274
30-35	**244**	246	243	245	244	229	247
35-40	**168**	167	163	166	157	150	164
40-45	**72**	72	79	75	77	86	76
45-49	**18**	18	13	14	30	45	15
10^6. SSQ		8.93	125	30.3	318	2006	59.7
K		5	4	5	4	3	6
AIC		**-76.2**	-62.6	-67.7	-56.1	-47.2	-56.1

Data source: http://www.measuredhs.com/pubs/pdf/FR148/04Chapter04.pdf , Table 4.1 p. 51.

Table 12: Empirical and fitted values for the ASFR for Rwanda with values of minimization and model selection criterion

Rwanda 2005							
Data		**Model Fit**					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**42**	42	42	59	50	63	41
20-25	**235**	235	235	222	224	215	238
25-30	**305**	304	304	307	316	314	301
30-35	**273**	275	274	285	278	285	277
35-40	**211**	209	211	202	190	193	206
40-45	**117**	118	120	110	112	108	118
45-49	**32**	32	22	43	59	54	35
10^6. SSQ		11.4	111	877	1523	1950	76.5
K		5	4	5	4	3	6
AIC		**-74.5**	-63.4	-44.1	-45.1	-47.4	-54.4

Data source: http://www.measuredhs.com/pubs/pdf/FR183/04Chapter04.pdf , Table 4.1 p. 38.

Table 13: Empirical and fitted values for the ASFR for Senegal with values of minimization and model selection criterion

Senegal 2005							
Data		**Model Fit**					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**101**	102	101	104	106	90	96
20-25	**212**	210	211	205	201	216	223
25-30	**250**	252	255	256	257	262	252
30-35	**228**	230	225	232	233	220	221
35-40	**169**	164	162	158	155	147	158
40-45	**74**	78	85	79	78	85	86
45-49	**22**	21	14	27	31	45	22
10^6. SSQ		69.4	278	266	518	1500	486
K		5	4	5	4	3	6
AIC		**-61.9**	-57.0	-52.5	-52.6	-49.2	-41.4

Data source: http://www.measuredhs.com/pubs/pdf/FR177/04Chapitre04.pdf , Table 4.1 p. 56.

Table 14: Empirical and fitted values for the ASFR for South Africa with values of minimization and model selection criterion

South Africa 2003							
Data		**Model Fit**					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**55**	55	55	55	59	54	56
20-25	**102**	100	101	100	94	101	99
25-30	**97**	101	101	102	104	104	102
30-35	**84**	82	80	82	83	78	82
35-40	**56**	53	53	52	50	48	52
40-45	**18**	22	25	24	23	26	24
45-49	**6**	4	4	4	8	13	4
10^6. SSQ		51.3	98.4	77.8	197	276	103
K		5	4	5	4	3	6
AIC		-64.0	**-64.3**	-61.1	-59.4	-61.1	-52.3

Data source: http://www.measuredhs.com/pubs/pdf/FR206/FR206.pdf , Table 3.1 p. 44.

Table 15: Empirical and fitted values for the ASFR for Zambia with values of minimization and model selection criterion

Zambia 2001-2002							
Data		Model Fit					
Age	**DHS**	**AEM**	**QS**	**Beta**	**Gamma**	**Hadwiger**	**Model 2**
15-20	**160**	160	159	163	170	158	164
20-25	**266**	261	267	257	244	261	253
25-30	**249**	257	250	259	265	267	262
30-35	**218**	219	218	220	225	212	221
35-40	**172**	160	165	159	152	144	157
40-45	**79**	89	91	89	84	89	89
45-49	**30**	27	15	27	39	52	29
10^6. SSQ		346	398	490	1386	1779	673
K		5	4	5	4	3	6
AIC		-50.6	**-54.5**	-48.2	-45.8	-48.0	-39.1

Data source: http://www.measuredhs.com/pubs/pdf/FR136/04Chapter04.pdf , Table 4.1 p. 56

Table 16: Parameter estimates of the AEM for selected countries

	Parameter				
Countries	**a**	**b**	**μ**	**S_1**	**S_2**
Cameroon	0.003	0.242	23.657	9.855	19.928
Ethiopia	0.008	0.247	25.485	10.338	20.066
Nigeria	0.008	0.267	27.553	13.746	16.097
South Africa	-0.001	0.106	23.702	9.512	19.889

Appendix B

Figure 2: Empirical and estimated ASFR for Cameroon (2004)

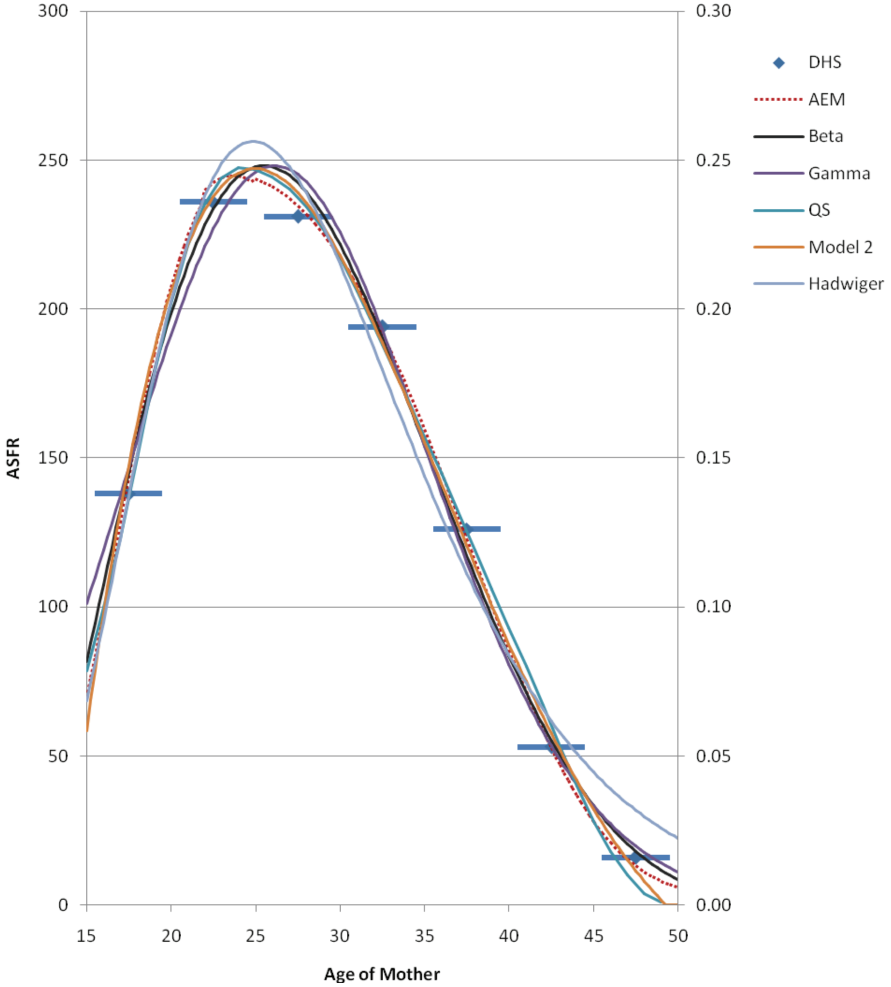

Figure 3: Empirical and estimated ASFR for Nigeria (2003)

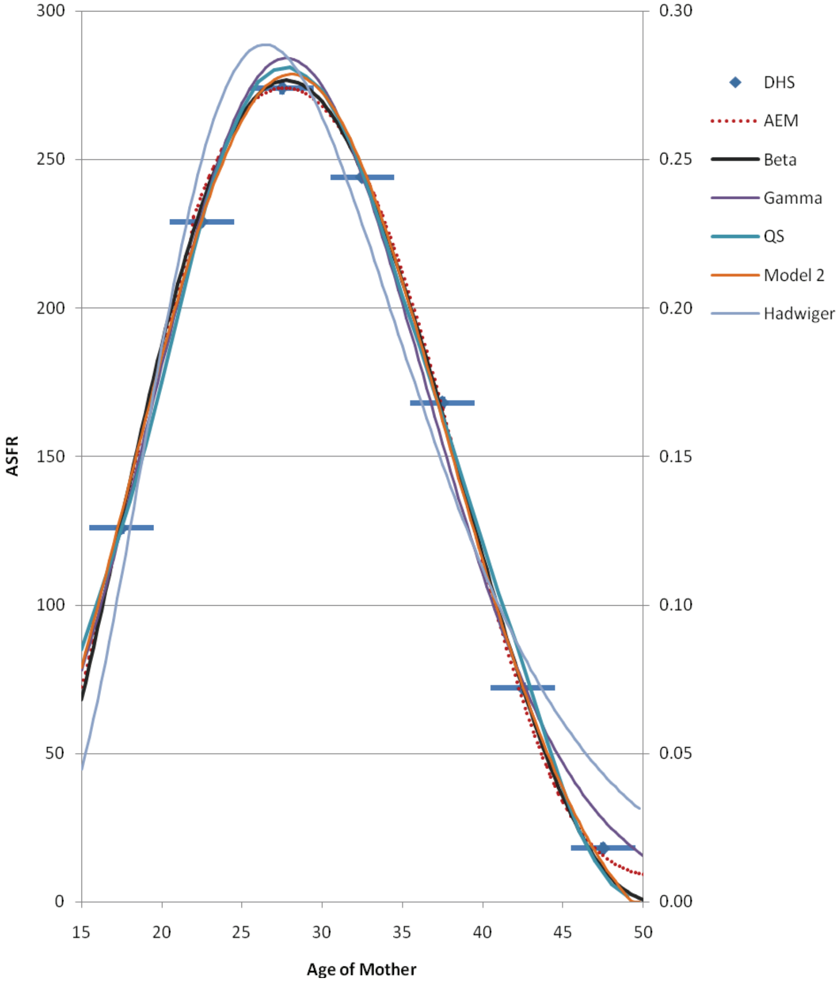

Figure 4: Empirical and estimated ASFR for South Africa (2003)

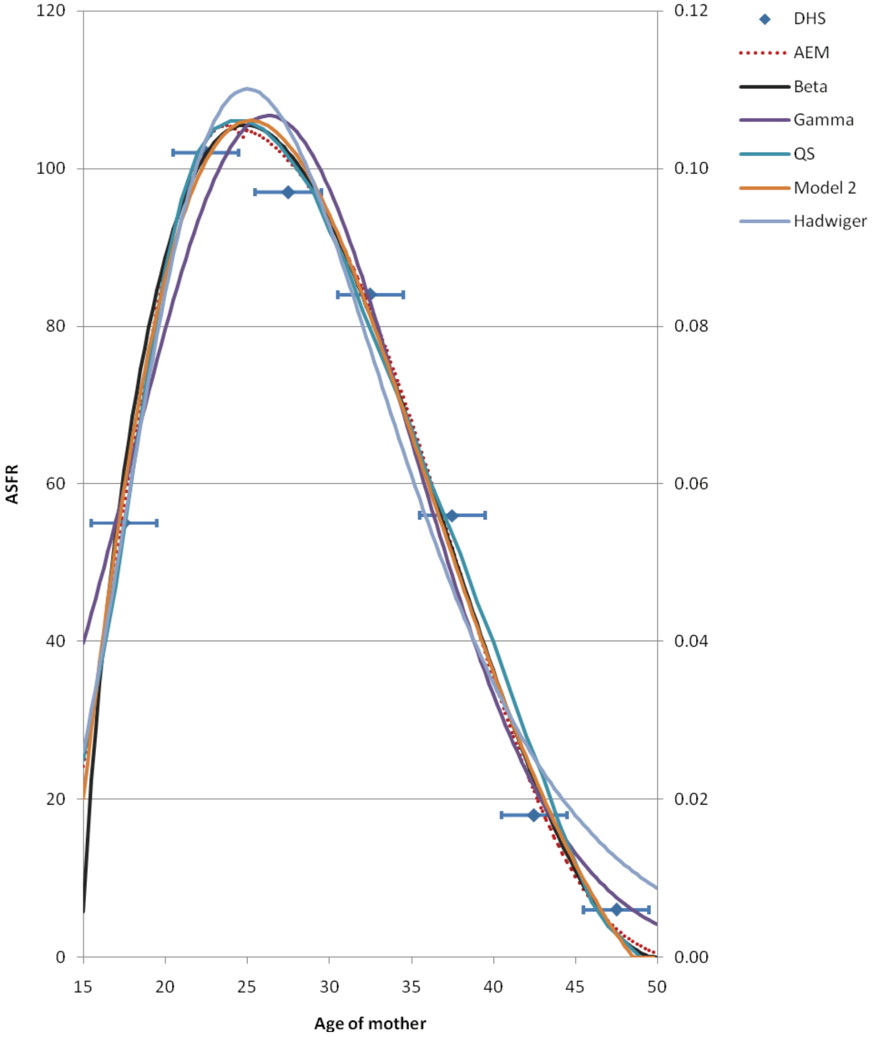

Figure 5: Relationship between modal fertility rate and parameter *b* of the AEM

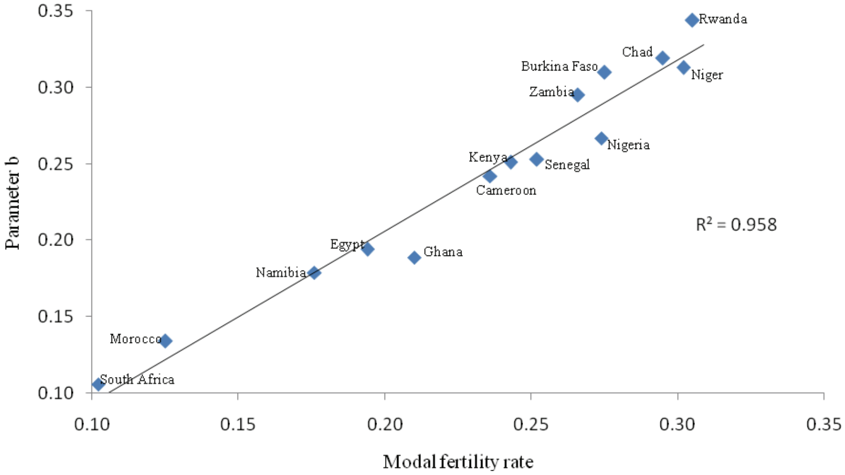

Figure 6: Relationship between fertility level at the age group 45-49 of the mother and parameter *a* of the AEM